

# Description of the Choquet Integral for Tactical Knowledge Representation

**Tod Schuck**  
Tactical C3I Systems  
Lockheed Martin MS2  
Moorestown, NJ  
[tod.m.schuck@lmco.com](mailto:tod.m.schuck@lmco.com)

**Erik Blasch**  
AFRL/RyAA Evaluation Branch  
Wright-Patterson AFB  
Dayton, OH  
[erik.blasch@wpafb.af.mil](mailto:erik.blasch@wpafb.af.mil)

**Abstract** - The goal of Combat Identification (CID), and as well, Situational Awareness (SA), is to combine data and information at the appropriate information representation in order to declare a positive ID according to a classification structure. CID includes the ultimate determination of the intent and prediction of future actions of an object or entity via the establishment of tactical knowledge. To facilitate CID, we utilize the concept of conceptual spaces to represent cooperative and non-cooperative CID. The Choquet integral combined with Bayes risk enables methods that provide a statistical approach to adversary intent prediction through the CID knowledge spaces. The use of the Choquet Integral for CID is applied in the context of a Maritime Domain Awareness (MDA) example.

**Keywords:** Conceptual spaces, Choquet integral, Combat ID, threat, intent determination.

## 1 Introduction

What is *tactical knowledge*? First knowledge can be defined and applied to situations where small scale knowledge (tactical information) relates to a larger purpose (strategic operations). One can then leverage a definition from Kessler and White [1] who utilized business logic from Davenport and Prusek [2] to provide a common reference frame. Therefore, knowledge can be defined as “a fluid mix of framed experience, values, contextual information, and expert insight that provides a framework for evaluating and incorporating new experiences and information”. One can note four themes of knowledge representation that includes information (what), context (when/where), evaluation (how), and expert users (who).

Knowledge representation may vary over new information and reasoning as well as the data representation which is important for tactical awareness. Difficulties arise over uncertain events. The Choquet integral can be useful for defining the likelihood of an uncertain event. Using the Choquet integral constructs for achieving tactical knowledge representation has these proposed benefits:

- Provides contextual stability and maintainability through additive capacities

- Provides contextual substance and meaning to new information
- Helps avoid contextual errors through top-down set based integration of information
- Provides confidence in decision-making through likelihood calculations
- Enables accurate and reliable decision-making over uncertain events
- Bounds the expectation through lower and upper threshold margins

Knowledge representation is ultimately a cognitive construct that a human achieves through a variety of mechanisms. Mostly, the human mind processes abstractions of information, often represented in some form of a spatial-temporal construct. The spatial-temporal approach to knowledge representation is discussed by Hawkins and Blakeslee in their recent book *On Intelligence* [3]. The human brain is not a number-crunching machine, rather Hawkins proposes that it stores memories, builds models of the world, and performs pattern associations. Maney [4] indicates that humans then use these models to make predictions based on a mental model. Each person utilizes their “tactical” model that forms a strategic understanding of the tactical domain.).

We propose that the usefulness of the Choquet integral, specifically the expansion provided by Warren [5], via cooperative, unintentionally cooperative, and non-cooperative object threat identification of moving targets, for intent assessment. Through the examples using the *Choquet Integration Function* (CIF), we show that a measure of information inconsistency between imprecise knowledge information can be determined and is related to a Bayes’ risk.

## 2 Conceptual Spaces

The context to place the idea of knowledge representation may be with the use of *conceptual spaces*. The concept of a conceptual space was pioneered by Gärdenfors [6]. Gärdenfors explains that there are two overarching goals in cognitive science. One is *explanatory* – the study of humans and animals to formulate theories of cognition. The other is *constructive* – the building of artifacts that can accomplish cognitive tasks (e.g. IBM’s Deep Blue supercomputer that played chess against Garry Kasparov).

For both explanatory and constructive goals, the issue is how the representations of knowledge (i.e. tactical constructs) are used by and modeled within a cognitive system.

Conceptual spaces are constructed from geometrical representations based on a number of quality dimensions that make up the space. As Gärdenfors illustrates for the conceptual space of taste, there are four quality dimensions (e.g. a four dimensional space) which includes salt, sweet, sour, and bitter. The taste dimensions are represented by the Henning taste tetrahedron in Figure 1 [7] as given by Gärdenfors.

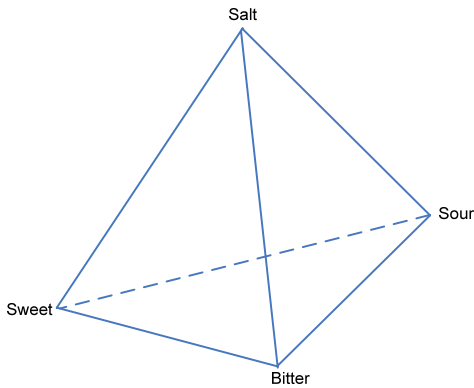


Figure 1. Quality Dimensions for Taste

Gärdenfors further posits that the conceptual space  $S$  can be established by a class of quality dimensions  $D_1, \dots, D_n$ ; where  $S$  is represented by a vector  $v$  equal to  $\langle d_1, \dots, d_n \rangle$ . Each dimension then has certain topological structure. The use of partitioning the conceptual space into dimensions is similar to the work of the Lorenz and Biermann [8], and Schuck and Hunter [9].

The equivalent diagram to the Henning Taste Tetrahedron for tactical information entities to identify a moving target from cooperative and non-cooperative ID sensing is shown in Figure 2.



Figure 2. Quality Dimensions for Combat Identification

These four quality dimensions consist of *kinematics*, *cooperative information* (e.g. Identification Friend-or-Foe (IFF)), *unintentionally cooperative information* (e.g. Electronic Support (ES)), and *non-cooperative information* (e.g. High Range Resolution (HRR) Radar). Kinematics is the envelope of possible object position states, velocities, and accelerations. Persistent IFF, such as used for air traffic control (ATC) is considered cooperative information because the information source willingly discloses information about itself to a requestor. ES is considered to be unintentionally cooperative information because the information source, in the course of its normal operations, unknowingly discloses information about its identity based on the characteristics of its emissions. Non-cooperative information (often termed Non-Cooperative Target Recognition (NCTR)) requires no cooperation from an information source other than its physical existence in order to derive features associated with its identity. The evidence from the CID quality dimensions can contribute to the original Joint Directors of Laboratories (JDL) model [10, 11, 12] definitions of *object refinement* of an object (F-16, tank, Airbus A300, small boat, etc.), the *situation refinement* of it (Friend, Hostile, Neutral, etc.), and ultimately the *threat refinement* (representing the knowledge of the object's intentions and capabilities) [13]. CID dimensions also support Data Fusion Information Group (DFIG) model Level 5 *user refinement* in target labeling [14].

### 3 Choquet Integral Constructs

Rickard [15] has generalized and extended the geometric theory of conceptual spaces to one that satisfies a *multi-dimensional fuzzy space*. In this fuzzy space reside domains that correspond to disjoint categories that define the “flavor” of information (Figure 2 represents this for CID/SA). For knowledge representation, Rickard states that “Objects in a conceptual space are represented by points, in each domain, that characterize their dimensional values” [15]. Within each domain we can look at the distance or similarity between objects as relative distances between points, which can be based on the aforementioned domain knowledge and fidelity of confidence. The natural application of fuzzy methods for abstract knowledge partitioning provides a transition to the concept of the Choquet integral.

#### 3.1 Choquet Origins

The Choquet integral is named for Gustave Choquet in his groundbreaking treatise the *Theory of Capacities* [16]. While an extensive set of work, Choquet's development focused on non-additive, subset functions that could define the likelihood of an uncertain event (a capacity). The Choquet integral is an early example of what was to become fuzzy logic as pioneered by Zadeh [17], with a capacity a type of fuzzy measure. The Choquet integral is a non-linear transformation that integrates a real function with respect to a fuzzy measure [18]. The Choquet relationship is given as [16]:

$$f(A \cup B) \leq f(A) + f(B) - f(A \cap B) \quad (1)$$

Where  $f$  is a capacity and  $A$  and  $B$  are subsets of a space  $X$ .

The Choquet relationship is built upon by Sugeno [19] in his classic work on the application of fuzzy integrals where for the same subsets  $A$  and  $B$  with  $A \cap B = \phi$ :

$$g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B) \quad (2)$$

Where  $g$  is the Sugeno measure (also referred to as a density) and  $\lambda$  is a constant on the interval  $[-1, \infty]$  that defines the additivity of the subsets, and is a probability measure when equal to 0 [18].

According to Klir [20], the Choquet integral is a kind of monotone measure,  $\mu$ , on the ordered pair  $[X, C]$  where  $X$  is the universal set and  $C$  is the nonempty family of subsets  $X$ . The Choquet is denoted as  $\mu: C \rightarrow [0, \infty]$  and satisfies for all  $A, B \in C$ , if  $A \subseteq B$ , then  $\mu(A) \leq \mu(B)$  (monotonicity).

If  $\mu(A \cup B)$  is either  $\geq$  or  $\leq \mu(A) + \mu(B) \forall A \cup B \in C$  such that  $A \cap B = \phi$ , then the monotone measure is called superadditive or subadditive respectively and its characteristics are expanded in the following way [20, 21]:

- a)  $\mu(A \cup B) > \mu(A) + \mu(B)$  synergy/cooperation between  $A$  and  $B$  where the importance of  $A$  and  $B$  together is greater than the sum of the individual importances – this is superadditivity
- b)  $\mu(A \cup B) = \mu(A) + \mu(B)$  ( $A$  and  $B$  are non-interactive and thus independent)
- c)  $\mu(A \cup B) < \mu(A) + \mu(B)$  (incompatibility between  $A$  and  $B$  where the importance of  $A$  and  $B$  together is less than the sum of the individual importances – this is subadditivity)

Klir [20] further states that classical probability theory can only capture (b), else the axiom of additivity is otherwise violated. Thus the theory of monotone measures like the Choquet Integral provides a richer framework for capturing and formalizing uncertainty.

Following these relationships (a-c), the *Choquet Integration Function* (CIF) is a non-additive fuzzy integral where subsets of information (or knowledge) are aggregated (not individual operators), which enables inter-element interdependencies (e.g. associations) and non-linearities to be captured. We can associate the values as a *distance measure* to capture the additive nature of knowledge representation. The Choquet Integral approach models the feature of human cognition which is concerned with the synthesis of information according to the *balance between information elements*, i.e. this provides a

mechanized *cognitive equilibrium* or *cognitive dissonance* method and can employ “negative” information processes, where the absence of information that is expected provides evidence towards an outcome.

### 3.2 Choquet Integration Function

The Choquet technique proposed by Warren [5] is a non-aggregation process that enables structures of knowledge variables to be developed for high level strategic processes even when interdependencies exist. Warren defines knowledge fusion as a higher abstracted process that sits above information and data fusion, which is consistent with our discussion of level 2 and 3 fusion in the Data Fusion Information Group (DFIG) model [14]. Warren has defined *Globular Knowledge Fusion* (GKF) as a method to synthesize information when interdependencies exist from inter-element causal influences through tiers that allow information to be shared in a non-additive fashion. In other words, the GKF method looks at the mapping of function values where individual values that are close together have an increasing effect on the aggregate value, but disparate values have a decreasing effect for superadditive weights [5]. Using causal relationships is ideal in the SA and CID realms because of the difficulty in establishing information interdependencies between multiple sources that may be related.

To incorporate knowledge elements in tactical awareness, the use of the CIF enables the representation of knowledge in a system to further define information aggregation. The discrete CIF ( $C(\bullet)$ ) is defined as [5]:

$$C(\bullet) = \sum_{i=1}^n (\mu(A_i) - \mu(A_{i-1})) f(x_i) \quad (3)$$

Where,

$$\lambda + 1 = \prod_{i=1}^n (1 + w_i \lambda) \quad (4)$$

$$u(A_i) = \mu(A_{i-1}) + w_i + \lambda w_i \mu(A_{i-1}) \quad (5)$$

With  $w_i$  = the individual information weights (as used in the Sugeno [19] equation (4)),  $u(A_i)$  = the monotone subset weight (where  $u(A_0) = 0$ ),  $\lambda$  = the non-additive parameter in the Sugeno equation, and  $f(x)$  is the global value estimate. Some of these measures (such as  $w_i$ ) could be determined from data mining techniques and knowledge of the reliability of information sources.

### 3.3 Choquet Simulations

To show the development and usefulness of the Choquet Integral for tactical knowledge representation, this section demonstrates simulations of the use of importance values for information additivity for knowledge representation. Assume an SA space of the relative function values of the cumulative cooperative ( $c_i$ ), unintentionally cooperative

( $u_i$ ), and non-cooperative ( $n_i$ ) information sets  $I$  (conceptual spaces) – shown in Figure 2 – completely define the information and thus knowledge space for CID and SA. Let  $u(A_i)$  represent the contributions from the three subsets ( $c_i, u_i, n_i$ ) from which the goal is to determine the kinematic  $k_i$  value of the object.

**EXAMPLE 1:** Marginal agreement between sources of tactical knowledge.

In the example, let ( $c_i = 63, u_i = 42, \text{ and } n_i = 21$ ). WAV is the Weighted Average of the three inputs.

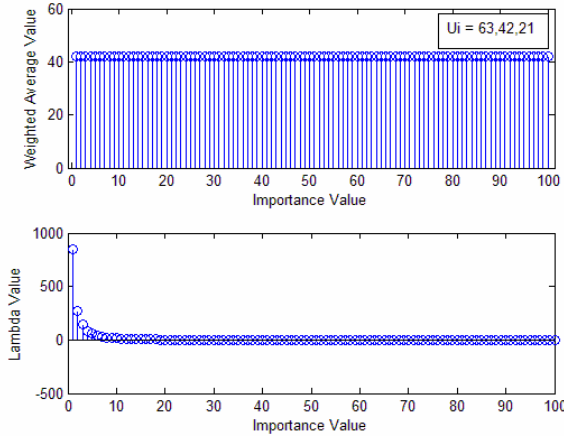


Figure 3. WAV and  $\lambda$  Values for  $w = 0.01$  to  $1.0$  for  $u(A_i) = 63, 42, 21$ .

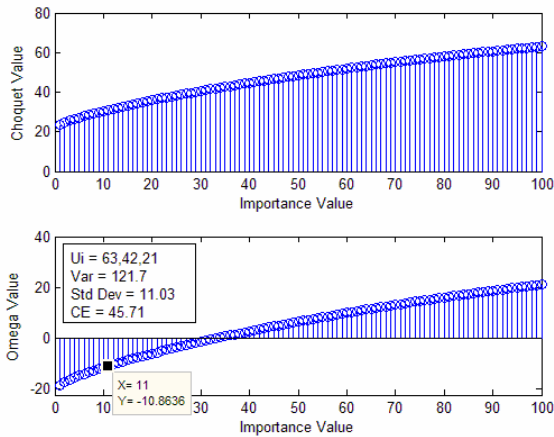


Figure 4. Choquet and  $\Omega$  Values for  $w = 0.01$  to  $1.0$  for  $\mu(A_i) = 63, 42, 21$ .

In the *MatLab* generated Figures 3 and 4, the *information weight*,  $w$ , is set across a range from  $0.01$  to  $1.0$  in  $0.01$  steps for three global values ( $f(x)$ ) of  $63, 42, \text{ and } 21$ , which equate to the subset weights  $u(A_i)$ . Any single value for  $w_i$  would reflect an expert opinion [22] of the value of the information if scaled to some normalized value. So the values for how these are set would have to follow experts

weighing in on the value of a given information source and source type. For example, a positive encrypted transponder reply would have a very high  $c_i$ .

A positive  $\lambda$  implies that individual global values  $f(x)$  have increasing value and synergy. The negative values model redundancy by decreasing the marginal strengths of increasing subset weights  $w_i$  [5]. The value of additive knowledge is achieved whenever the sum of for all inputs is greater than one (e.g. when the sample number exceeds 33) which is the point where  $\lambda$  becomes negative per the interval  $[0, -1]$ . Using a value of information may help to determine when information lacks independence. For example, there may be some information overlap between communications intelligence (COMINT) (one type of unintentionally cooperative information) and other information. The aggregation process can then have designed non-linearities that depend upon the degree of consistency of the data. The  $u(A_i)$  are always bounded and range from  $0$  to  $100$  which could be provided from the results of a lower order fusion process such as probabilities from a Bayesian or evidential process. The  $\Omega$  value is simply the weighted average (WAV) value of the subset weights minus the Choquet value which is a measure of *information inconsistency* [5]. Figure 4 also shows the results of the statistical analysis of the *standard deviation* (Std Dev), *variance* (Var), and *certainty equivalent* (CE). The determination and meaning of the CE is discussed in section 4.

**EXAMPLE 2:** Disagreement between sources of tactical knowledge.

Further insight on the behavior of the Choquet function for other subset weights follows ( $c_i = 83, u_i = 42, \text{ and } n_i = 1$ ).

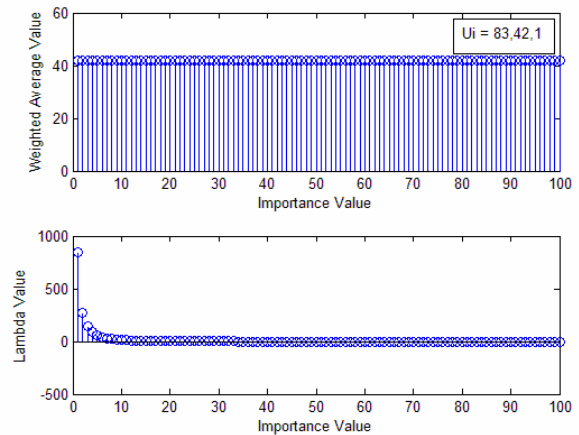


Figure 5. WAV and  $\lambda$  Values for  $w = 0.01$  to  $1.0$  for  $\mu(A_i) = 83, 42, 1$ .

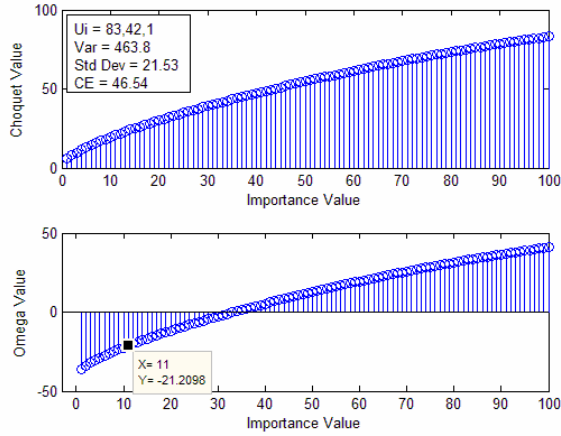


Figure 6. Choquet and  $\Omega$  Values for  $w = 0.01$  to  $1.0$  for  $\mu(A_i)=83,42,1$

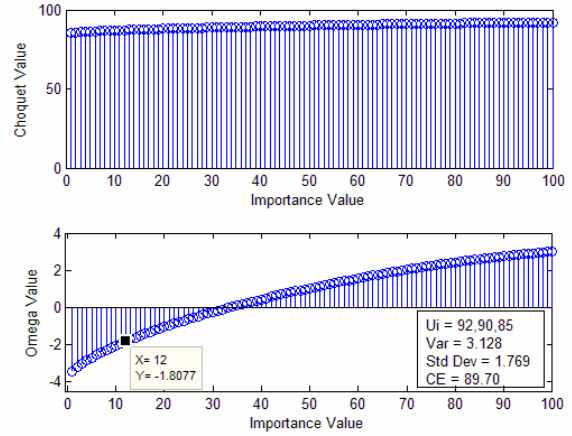


Figure 8. Choquet and  $\Omega$  Values for  $w = 0.01$  to  $1.0$  for  $\mu(A_i) = 92, 90, 85$

The divergent subset values of  $(c_i, u_i, n_i)$  used in the generation of Figures 5 and 6 shows that the information inconsistency,  $\Omega$ , has a dramatic effect on the change in the Choquet values. The difference in the initial  $\Omega$  values are about double even though the WAV is the same for both sets of  $u(A_i)$ . Regardless, once the  $w_i$  values are greater than  $0.33$ , the omega values are about the same until the greater values of  $w_i$  show superadditivity and increase to larger values than in example 1 because of the increased disparity in values.

**EXAMPLE 3:** Agreement between tactical knowledge.

Continued insight on the behavior of the Choquet function for other subset weights follows ( $c_i = 92, u_i = 90, n_i = 85$ ).

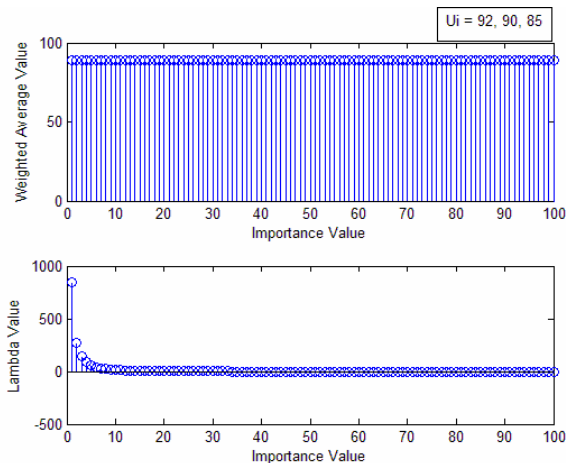


Figure 7. WAV and  $\lambda$  Values for  $w = 0.01$  to  $1.0$  for  $\mu(A_i)=92, 90, 85$

In Figures 7 and 8 it can be seen that when the subset weights are closely spaced (in agreement) with  $(c_i = 92, u_i = 90, n_i = 85)$  the Choquet values are only minimally different from the WAV with a small variance. Here, regardless of the importance weight value, a decision could be made with high confidence.

In terms of confidence in decision making, in Figures 4, 6, and 8, as  $w_i$  increases, the Choquet function values increase. This leads to a decrease in information inconsistency and could represent a tactical knowledge increase. Thus, if this knowledge state  $(c_i, u_i, n_i)$  was established at an automated decision process, then there might be cause to then report out this knowledge state based on some sort of decision threshold. Part of the challenge of reporting out and requesting information and knowledge is the necessity to determine when more information is needed and establishing decision threshold.

**4 Defining the CIF as a Bayes Risk Function**

A Bayes risk function can aid in the decision threshold analysis. The most common risk function used for Bayesian estimation is the mean square error (MSE), also called *squared error risk*. The MSE is defined by the following,

$$MSE = E[(\hat{\theta}(x) - \theta)^2] \tag{6}$$

where the expectation is taken over the joint distribution of  $\theta$  and  $x$ .

Using the MSE as risk, the Bayes estimate of the unknown parameter is simply the mean of the posterior distribution,

$$\hat{\theta}(x) = E[\theta|X] = \int \theta f(\theta|x) d\theta \tag{7}$$

This is known as the *minimum mean square error* (MMSE) estimator. The Bayes risk, in this case, is the *posterior variance*<sup>1</sup> [22]

For the case of our Choquet calculations, the posterior variance is available from our previous calculations of a given set of three global values across a range of importance values. In Figures 4, 6, and 8, the variance and standard deviation of the posterior Choquet values are determined. An interesting characteristic of the Choquet values is that regardless of the global value values or their relative agreement, the first standard deviation from the zero disagreement point ( $\Omega = 0$  when all  $w_i = 0.33$  (where  $\lambda$  becomes negative)) *occurs* at about the importance value of 0.11 (between 0.09 and 0.12). The mean, standard deviation, and CE values are shown in the box in all three figures. For a risk-averse decision maker, the CE is less than the expected value of the uncertain “gamble” in the decision space because the decision-maker wants to reduce uncertainty. The CE calculation is discussed in Clemen and Reilly [24] from the approximation by Pratt [25] when the expected value (mean) and variance are available and is defined as

$$CE \approx \mu - \frac{0.5\sigma^2}{RT} \quad (8)$$

where  $\mu$  is the mean of the distribution (not to be confused with  $\mu(A_i)$  which is the monotone subset weight),  $\sigma^2$  is the variance and  $RT$  is the *risk tolerance*. For the results in Figures 4, 6, and 8, the  $RT$  was set to the Choquet value at the zero disagreement point at  $w_i = 0.33$  for each distribution.

For each set of subset weights discussed, the CE is always less than  $\mu$ . However, as the agreement between the subset weights becomes perfect (when they are all equal), the CE is equivalent to  $\mu$ . So a decision can be made based on the risk assessment of how far from complete agreement or knowledge is sufficient. In essence, with the monotone stationary statistics of the Choquet values, risk tolerance can be readily mapped to decision outcomes of a knowledge state. Like Bayes risk, the CE is an estimator of the MSE.

## 5 Conclusions and Recommendations

In this paper we have presented a means through the use of the fuzzy Choquet integral to represent knowledge for use in tactical decisions. Our basis of knowledge instantiation is based on the research of how human decision-making is performed. Since human decision making is not based on pure number crunching, but rather considers a more abstract weighting of information sets, our approach based on the discrete Choquet integral as described by Warren [5] attempts to mimic a user-

abstracted weighted process. In other words, the method presented using the Choquet integral supports tactical knowledge-based decision needs.

Some mention of the “ugly truth” associated with lower level information fusion needs to be considered. As described by Hall and Llinas [26] in their classic work on the ugly truths in multi-sensor data fusion, if the underlying data is problematic or poorly understood, then there is little chance of crafting a meaningful fusion process. Generating a reliable process that the user can trust is certainly true for knowledge-based fusion. Well profiled sensors and data providers with a common ontology feeding a robust, well-understood probabilistic or evidential fusion method are required to make our proposed Choquet knowledge fusion process work. Otherwise, no amount of numerical alchemy will provide useful outputs and may even cloud the understanding of a situation, especially a tactical one.

Further work will extend the analysis to a more complete representation of a specific tactical problem for homeland security or military application. We have completed additional material that will be published that considers hypotheses from a series of observations were made of a small fishing boat transiting a littoral area off of the northeast coast of the United States for Maritime Domain Awareness (MDA). Some hypotheses that our model Threat Discernment system will try to answer include: “The trawler ship *Valerie Day* is preparing to fish in a prohibited fishing zone.”, “The trawler ship *Valerie Day* is transiting through the region” and “The trawler ship *Valerie Day* is transporting contraband cargo”. Because of our knowledge of the area and activities in this region, the subset weight assignment intervals are mapped to the complete list of hypotheses and subset weights from cooperative, unintentionally cooperative, and non-cooperative systems.

Additional domains of application could come from areas like border control, terrorist target analysis, and mixed military/peacekeeping operations common in Iraq and Afghanistan. Also, the use of non-equal importance weights should be undertaken since differing values of information are closer to what is representative for realized sensor systems. These could be based on understanding of individual sensor systems and characteristics of their fused results. Our additional work will demonstrate a closed form solution for differing individual values of  $w_i$ .

Generally cooperative systems provide rich information sets, but unless the integrity of the information can be assured, there is the question of corruption by intentional deception. Unintentionally cooperative and non-cooperative can provide more definitive information, but it is usually harder to get and can involve high “expense” and time investments such as the acquisition of satellite

<sup>1</sup> [http://en.wikipedia.org/wiki/Bayes\\_estimator](http://en.wikipedia.org/wiki/Bayes_estimator).

imagery. Understanding these real-world issues will lead to well-described subset weights and importance values.

## References

- [1] O. Kessler and F. White, *Data fusion perspectives and its role in information processing*, Handbook of Multisensor Data Fusion, Second Edition: Theory and Practice M. Liggins, J. Llinas and D. Hall (eds.) Boca Raton, FL: Taylor and Francis, 2008, 15—44.
- [2] T. Davenport, and L. Prusek, *Working knowledge: how organizations manage what they know*, Harvard Business School Press, Boston, MA, 2000.
- [3] J. Hawkins and S. Blakeslee, *On Intelligence*, New York, NY: Times Books, 2004.
- [4] K. Maney, *Father of Palm handhelds focuses on making computers even brainier*, USA Today, March 29, 2005.  
[http://www.usatoday.com/money/industries/technology/maney/2005-03-29-palm\\_x.htm](http://www.usatoday.com/money/industries/technology/maney/2005-03-29-palm_x.htm)
- [5] L. H. Warren, *Strategic information synthesis by globular knowledge fusion*, Proceedings of Information, Decision and Control, February 8-10, 1999, 407—412.
- [6] P. Gärdenfors, *Conceptual spaces as a framework for knowledge representation*, Mind and Matter, vol. 2(2), 2004, 9—27.
- [7] H. Henning, *Die qualitatensreihe des geschmacks*, Zeitschrift für Psychologie und Physiologie der Sinnesorgane, 1961, 203—219.
- [8] F. P. Lorenz and J. Biermann, *Knowledge-based fusion of formats: discussion of an example*, Proceedings of the 5th International Conference on Information Fusion, vol. 1, July 8-11, 2002, 374—379.
- [9] T. Schuck and J. Hunter, *Distributed classification in a multi-source environment*, Proceedings of the Fifth Annual Conference on Information Fusion, vol. 2, 2003, 874—880.
- [10] J. Llinas, C. Bowman, G. Rogova, A. Steinberg, E. Waltz and F. White, *Revisiting the JDL data fusion model II*, Proceedings of the 7th International Conference on Information Fusion, 2004.
- [11] A. Steinberg, C. Bowman and F. White, *Revisions to the JDL data fusion model*, NATO/IRIS Conference, October, 1998.
- [12] F. E. White Jr., *Data fusion subpanel report*, Technical Proceedings of the 1991 Joint Service Data Fusion Symposium, vol. 1, Laurel, MD, 1991.
- [13] F. E. White Jr., *A model for data fusion*, Proceedings of the 1st National Symposium on Sensor Fusion, 1988.
- [14] E. Blasch, I. Kadar, J. Salerno, M. M. Kokar, S. Das, G. M. Powell, D. D. Corkill, and E. H. Ruspini, *Issues and challenges of knowledge representation and reasoning methods in situation assessment (Level 2 Fusion)*, J. of Advances in Information Fusion, Dec. 2006.
- [15] J. Rickard, *A concept geometry for conceptual spaces*, Fuzzy Optimum Decision Making, vol. 5, 2006, 311—329.
- [16] G. Choquet, *Theory of capacities*, Annales de l'Institut Fourier, vol. 5, 1953, 131—295.
- [17] L. Zadeh, *Fuzzy sets*, Control 8, 1965, 338—353.
- [18] P. Gader, A. Mendez-Vasquez, K. Chamberlin, J. Bolton and A. Zare, *Multi-sensor and algorithm fusion with the Choquet integral: applications to landmine detection*, IEEE International Geoscience and Remote Sensing Symposium, vol. 3, September 2004, 1605—1608.
- [19] M. Sugeno, *Theory of fuzzy integrals and its applications*, Tokyo Institute of Technology, PhD Thesis, 1974.
- [20] G. Klir, *Uncertainty and Information, Foundations of Generalized Information Theory*, Hoboken, NJ: John Wiley and Sons, Inc., 2006.
- [21] M. Grabisch, *A graphical interpretation of the Choquet integral*, IEEE Transactions on Fuzzy Systems, vol. 8, no. 5, October 2000, 627—631.
- [22] R. Hummel and M. Landy, *A statistical viewpoint on the theory of evidence*, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 10, no. 2, March 1988, 235—247.
- [23] D. Johnson, *Minimum mean squared error estimators*, Connexions, 2004.  
(<http://cnx.org/content/m11267/latest/>)
- [24] R. Clemen and T. Reilly, *Making Hard Decisions with Decision Tools*, Pacific Grove, CA: Duxbury Thompson Learning, 2001.
- [25] J. Pratt, *Risk aversion in the small and in the large*, Econometrica, vol. 32, 1964, 122—136.
- [26] D. Hall and J. Llinas, *Multisensor data fusion*, Handbook of Multisensor Data Fusion, Second Edition: Theory and Practice, M. Liggins, J. Llinas and D. Hall (eds.), Boca Raton, FL: Taylor and Francis, 2008, 1—14.